MATH 2E Prep: Partial Derivatives

1. Find the first partial derivatives of the function $f(x, y) = \frac{x}{y}$, then find a point $P(a, b)$ such that

$$\frac{\partial f}{\partial x}(a, b) = \frac{\partial f}{\partial y}(a, b) = \frac{1}{2}.$$

Solution: Treat y as constant, get $\frac{\partial f}{\partial x} = \frac{1}{y}$.
Treat x as constant, get $\frac{\partial f}{\partial y} = -\frac{x}{y^2}$.
$\frac{1}{2} = \frac{\partial f}{\partial x} = \frac{1}{y} \Rightarrow y = 2$.
$\left\{ y = 2 \text{ and } \frac{1}{2} = \frac{\partial f}{\partial y} = -\frac{x}{y^2} \right\} \Rightarrow x = -2$.
So the point is $(-2, 2)$.

2. Find the first partial derivatives of the function $w = \ln(x + 2y + 3z)$.

Solution: Treat y and z as constant, get $\frac{\partial w}{\partial x} = \frac{1}{x + 2y + 3z}$.
Treat x and z as constant, get $\frac{\partial w}{\partial y} = \frac{2}{x + 2y + 3z}$.
Treat x and y as constant, get $\frac{\partial w}{\partial z} = \frac{3}{x + 2y + 3z}$.

3. Find the gradient of the function $f(x, y, z) = \frac{xz}{x^2 + y^2}$, and evaluate the gradient at the point $Q = (1, 1, 0)$

Solution:

$$\frac{\partial f}{\partial x} = \frac{z(x^2 + y^2) - xz(2x)}{(x^2 + y^2)^2} = \frac{z(y^2 - x^2)}{(x^2 + y^2)^2}$$
$$\frac{\partial f}{\partial y} = \frac{-xz(2y)}{(x^2 + y^2)^2} = \frac{-2xyz}{(x^2 + y^2)^2}$$
$$\frac{\partial f}{\partial z} = \frac{x}{x^2 + y^2}$$

So

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle = \left\langle \frac{z(y^2 - x^2)}{(x^2 + y^2)^2}, \frac{-2xyz}{(x^2 + y^2)^2}, \frac{x}{x^2 + y^2} \right\rangle$$

Evaluating at $Q = (1, 1, 0)$, we get

$$\nabla f(1, 1, 0) = \langle 0, 0, \tfrac{1}{2} \rangle$$